

Modeling and Approximation of an Incremental Fuel Cost-Curve for Generation Expansion Planning

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ABSTRACT

The tendency to generate electricity at the lowest cost for economic dispatch program especially to the electric utilities have become a major challenges, thus this problems will strongly affect the activities and operations in the power industry. This activity imposes a complex and complicated equation to be solved. Therefore Modelling and approximation equations are formulated and presented, to solve the problems of fuel-cost-minimization. The model equation strongly recognizes the cubic-polynomial behavior of fuel-cost, whether the function is increasing or decreasing similarly at what point this points are non-monotonically increasing or decreasing. This model will provide an efficient distribution of load capacity for an optimal power generation with the aim of minimizing cost. This model approximation will seriously help both to the system operations and planners. The demonstration and analysis of the model reveals strongly the needs to monitor fuel consumption capacity of generators in a way to make a saving, in order to drive the economy efficiency at large.

Keyword: Modeling, Approximation, Incremental Fuel, Cost-Curve, Generation Expansion Planning

1.1 Introduction

In power generation expansion planning, electric utilities have an encountered the major challenges in modeling and approximating the fuel cost pattern, in a way to determine the fuel minimization program to save cost. The model approach for approximation depends on economic dispatch technique and therefore required the information of the incremental cost curve. The paper consider the behavior of the curve either monotonically or non- monotonically increasing or decreasing function, thereby considering the number of linear segments and their corresponding break-points are determined. The fuel consumption coefficient and determined using multiple linear regression for the cubic fuel cost input output (I/O) curve. The paper recognizes that at minimum cost of operating point, the incremental cost for all the generating units are equal (power balance equation); this is the equality constraint conditions if we have for one or more generation units, and the energy balance equation is violated, then we look at the inequality constrain, where the optimum strategy is obtained by keeping these generation unit in their nearest limits and making the other generator units to supply the remaining power as per equal incremental cost - rule.

Fuel cost minimization approach requires knowledge of the fuel cost-curves for each of the generating units. An accurate representations of the cost-curve may requires a piece wise polynomial form, or can be approximated in several ways with common ones being:

- piece wise linear
- quadratic
- cubic
- piece wise quadratic

The linear approximation is not commonly used, which is the piece wise linear form that is used in many production - grade of linear programming applications. While a quadratic approximation is used in most non-linear programming application. Hence, control variable are usually independable variable in an Optimal Power Flow problems (OPF), including:

- active power generation
- generation bus voltages
- Transformer tap ratios
- phase - shifter angles
- values of switchable shunt capacitors and inductors.

The major cost of plant operations is fuel. Other cost may be added to the fuel-cost usually the fuel

cost is in \$/hr or ₹/hr and this is a function of power generation in MW. The fuel cost-curve may be assumed to be: linear, quadratic, parabolic, cubic function etc. But in the case of we may consider a parabolic form:

$$C_1 = \alpha_i + \beta_i p_i + \gamma_i p_i^2 \quad (1)$$

The incremental fuel-cost curve or slope of the fuel-cost curve is defined by:

$$\frac{dc_i}{dp_i} = 2\gamma_i p_i + \beta_i \quad (2)$$

The incremental fuel -cost curve, indicates how, expensive it will be to generate the next increment of power generation.

where:

$$C_1 = \$/hr \text{ or } ₹/hr$$

$$\frac{dc_i}{dp_i} = \$/hr \text{ or } ₹/MWh$$

2. Analysis and Model

The analysis and principle of equal incremental fuel cost is the coordination equation as:

$$f_i = P f_i \times p G_i$$

or

$$\frac{df_i}{dp G_i} = P F_i = \lambda_{system}; \quad i = 1, 2, 3, \dots, n \quad (1)$$

and

$$\sum P G_i = P_D + P_L \quad (2)$$

where:

f_i = Fuel-cost (l/0) for function unit 'i' in BTu/Hr or \$/Hr or ₹/Hr.

This function, (f_i) is represented by either a second or third order polynomial with respect to $P G_i$.

$$\frac{df_i}{dP G_i} = \text{incremental fuel-cost function (IC) for unit 'i' in BTu/Hr or } \$/MWh \text{ or } ₹/MWh$$

$P G_i$ = power - output of unit 'i' in MW.

$P F_i$ = transmission loss penalty loss factor for unit 'i'.

λ_{system} = system incremental fuel-cost of delivered power in BTu/MWh or \$/MWh or ₹/ MWh .

N = number of unit being dispatched.

P_D = total load, in MW.

P_L =transmission losses in MW.

Modeling of generator fuel-cost curves appear more fulfil in a third -order polynomial equation; even in practice.

That is,

$$f_i = a_{0i} + a_{1i} P G_i + a_{2i} P G_i^2 + a_{3i} P G_i^3 \quad (3)$$

differentiating the function, with respect to $P G_i$ we have as:

$$\frac{df_i}{dp G_i} = IC = a_{1i} + 2a_{2i} P G_i + 3a_{3i} P G_i^2 \quad (4)$$

Assuming, that equation (4) , is continuous function throughout the range from minimum to maximum operating limits, as shown in figure 1.0.

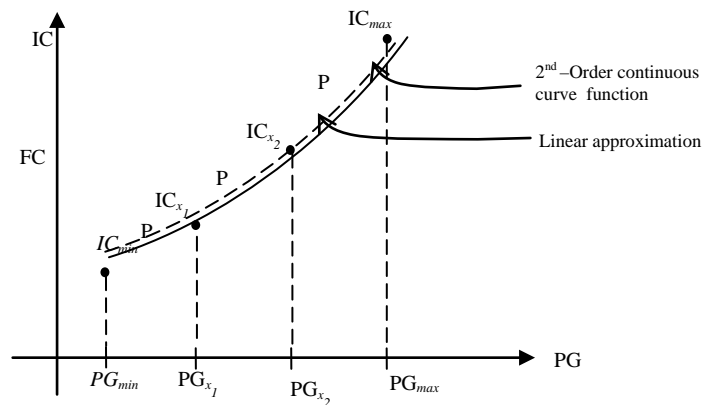


Fig.1 incremental cost (IC) curve versus generation, ($P G_i$)

- The idea or how best to approximate this continuous - functions, rely on linear-curve, in order to maintain a best optimality.

- The analysis of generator input-output (I/0) test data results in a way to determine both the number and placement of break-points on the continuous function.
- Therefore, there is strong need to determine the coefficients: a_{0i}, a_{2i}, a_{3i} respectively such that the IC will always be monotonically increasing.
- optimal placement of break-points or the continuous function requires:
 - ✓ Using generator I/0 test data from numerous units.
 - ✓ Multiple linear regression of third order polynomial can be used comfortably to represent, the fuel-cost (I/0) curve.
 - ✓ This will result to 2nd - order IC curve.
 - ✓ Similarly, if a second-order-fuel cost is assumed, then the IC curve will be linear (that is 1st - order).
 - ✓ Three (3) points are identified as break-points on the continuous-curve function, at: $(PG_{min}, IC_{min}), (PG_x, IC_x), (PG_y, IC_y)$ and (PG_{max}, IC_{max}) .
 - ✓ The continuous - curve function of second order IC curve, can be approximated by linear-segmentation.
 - ✓ Therefore, the IC curve of fig.1.0 can be approximated by three (3) linear segments with break-points at $[PG_{min}, IC_{min}], [PG_x, IC_x], [PG_y, IC_y]$ and $[PG_{max}, IC_{max}]$.
 - ✓ It is also assumed that, PG_y and IC_y lies on the 2nd -order continuous curve function
 - ✓ Another consideration, is that what should be the placement, such that the error between the linear approximation curve and the 2nd - order continuous curve becomes 'minimized'?
 - ✓ Thus, there is need to determine the error-function (V) as defined as:

$$V = \Delta A_1^2 + \Delta A_2^2 + \Delta A_3^2 \quad (5)$$

where:

ΔA_1 = area under linear curve from PG_{min} to minus PG_x area under the continuous - 2nd - order curve over the same range (PG_{min} to PG_x)

ΔA_2 = area under the linear curve from PG_{min} to PG_x minus area under the continuous - 2nd - order curve but over the range from PG_x to PG_y .

ΔA_3 = Area under the linear curve and 2nd - order curve - function over the range from PG_y to PG_{max} .

Case 1: 1st Break-point:

- ✦ Let A_1 be area under the linear-curve, from PG_{min} to PG_x and A_1^1 be the area under the continuous curve for the same range

That is, form $PG_{min} \rightarrow PG_x$ (pt1) from $PG_x \rightarrow PG_y$ (pt2), from, $PG_y \rightarrow PG_{max}$ (pt3) respectively

$$A_1 = \left(\frac{Ic_{min} + Ic_x}{2} \right) (PG_x - PG_{min}) \quad (7)$$

or

$$A_1 = \left(\frac{Ic_{min}}{2} + \frac{Ic_x}{2} \right) (PG_x - PG_{min}) \quad (8)$$

$$A_1 = \frac{Ic_{min}}{2} PG_x + \frac{Ic_x}{2} PG_x - \frac{Ic_{min}}{2} PG_{min} - \frac{Ic_x}{2} PG_{min} \quad (9)$$

Similarly,

$$A_1^1 = a_1 pg + a_2 pg^2 + a_3 pg^3 \quad (10)$$

or

$$A_1^1 = a_1 (PG_x - PG_{min}) + a_2 (PG_x^2 - PG_{min}^2) + (PG_x^3 - PG_{min}^3) \quad (11)$$

recalled equation (4) :

$$\frac{df_i}{dpG_i} = IC_i = a_{1i} + 2a_{2i} PG_i + 3a_{3i} PG_i^2 \quad (4)$$

That is, IC_{min} and IC_x in equation (7) are substituted into equation (4):

$$A_1 = \left[\frac{IC_{\min} + IC_x}{2} \right] (PG_x - PG_{\min}) \quad (7)$$

or

$$A_1 = \left[\frac{\left(a_1 + 2a_2 PG_{\min} + 3a_3 PG_{\min}^2 \right) + \left(a_1 + 2a_2 PG_x + 3a_3 PG_x^2 \right)}{2} \right] \quad (12)$$

$$(PG_x - PG_{\min})$$

or

- expanding equation (12) we have as:

$$A_1 = \left[\frac{IC_{\min} + IC_x}{2} \right] (PG_x - PG_{\min}) \quad (7)$$

$$A_1 = \left[\frac{\left(a_1 + 2a_2 PG_{\min} + 3a_3 PG_{\min}^2 \right) + \left(a_1 + 2a_2 PG_x + 3a_3 PG_x^2 \right)}{2} \right] \quad (12)$$

$$(PG_x - PG_{\min})$$

or

$$A_1 = \frac{a_1 PG_x + 2a_2 PG_{\min} \cdot PG_x + 3a_3 PG_{\min}^2 \cdot PG_x + a_1 PG_x + 2a_2 PG_x \cdot PG_x + 3a_3 \cdot PG_x^2 \cdot PG_x -}{2}$$

$$\frac{a_1 PG_{\min} - 2a_2 PG_{\min} \cdot PG_{\min} - 3a_3 PG_{\min}^2 \cdot PG_{\min} - a_1 \cdot PG_{\min} - 2a_2 PG_x \cdot PG_{\min} - 3a_3 \cdot PG_x^2 \cdot PG_{\min}}{2}$$

or

$$A_1 = \frac{a_1 PG_x + a_1 PG_x + 2a_2 PG_x PG_x + 3a_3 PG_{\min}^2 \cdot PG_x + PG_x + 3a_3 PG_x^2 \cdot PG_x -}{2}$$

$$\frac{a_1 PG_{\min} - a_1 PG_{\min} - 2a_2 PG_{\min} \cdot PG_{\min} - 3a_3 PG_{\min}^3 - 3a_3 PG_x^2 \cdot PG_{\min}}{2} \quad (13)$$

or

$$A_1 = \frac{2a_1 PG_x + 2a_2 PG_x PG_x + 3a_3 PG_{\min}^2 PG_x + 3a_3 PG_x^2 \cdot PG_x -}{2}$$

$$\frac{2a_1 PG_{\min} - 2a_2 PG_{\min} \cdot PG_{\min} - 3a_3 PG_{\min}^3 - 3a_3 PG_x^2 \cdot PG_m}{2} \quad (14)$$

or

$$A_1 = a_1 (PG_x - PG_{\min}) + a_2 (PG_x^2 - PG_{\min}^2) + \frac{3a_3}{2} (PG_x^3 - PG_x^2 PG_{\min} - PG_{\min}^3 + PG_{\min}^2 PG_x) \quad (15)$$

Recalled equation (11) and (15), the subtracting (11) from equation (15) we have:

$$A_1 = \frac{a_1 (PG_x - PG_{\min}) + a_2 (PG_x^2 - PG_{\min}^2) + 3a_3 (PG_x^3 - PG_x^2 PG_{\min} - PG_{\min}^2 + PG_{\min}^2 PG_x)}{2} \quad (15)$$

$$A_1^1 = a_1 (PG_x - PG_{\min}) + a_2 (PG_x^2 - PG_{\min}^2) + a_3 (PG_x^3 - PG_{\min}^3) \quad (11)$$

subtract and simplifying further:

$$\Delta A_1 = A_1 - A_1^1 = a_1 (PG_x - PG_{\min}) - a_1 (PG_x - PG_{\min}) + a_2 (PG_x^2 - PG_{\min}^2) - a_2 (PG_x^2 - PG_{\min}^2) +$$

$$\frac{3a_3}{2} (PG_x^3 - PG_x^2 PG_{\min} - PG_{\min}^3 + PG_{\min}^2 PG_x) - a_3 (PG_x^3 - PG_{\min}^3) \quad (16)$$

or

$$\Delta A_1 = A_1 - A_1^1 = \frac{3a_3}{2} PG_x^3 - \frac{3a_3}{2} PG_x^2 PG_{\min} - \frac{3a_3}{2} PG_x^3 + \frac{3a_3}{2} PG_{\min}^2 PG_x - a_3 PG_x^3 + a_3 PG_{\min}^3 \quad (17)$$

Collecting like terms

$$\begin{aligned} \Delta A_1 &= A_1 - A_1^1 = \frac{3a_3}{2} PG_x^3 - \frac{a_3}{1} PG_x^3 - \frac{3a_3}{2} PG_x^2 PG_{\min} \\ &\quad - \frac{3a_3 PG_{\min}^3}{2} + a_3 PG_{\min}^3 + \frac{3a_3}{2} PG_{\min}^2 PG_x \end{aligned} \quad (18)$$

$$\begin{aligned} \Delta A_1 &= \left(\frac{3a_3}{2} PG_x^3 - \frac{a_3 PG_x^3}{1} \right) + \left(\frac{-3a_3 PG_{\min}^3}{2} + \frac{a_3 PG_{\min}^3}{1} \right) \\ &\quad + \frac{-3a_3}{2} PG_x^2 PG_{\min} + \frac{3a_3}{2} PG_{\min}^2 PG_x \end{aligned} \quad (19)$$

or

$$\begin{aligned} \Delta A_1 &= \frac{a_3 PG_x^3}{2} - \frac{a_3 PG_{\min}^3}{2} - \frac{3a_3}{2} PG_x^2 PG_{\min} \\ &\quad + \frac{3a_3}{2} PG_{\min}^2 PG_x \end{aligned} \quad (20)$$

or

$$\Delta A_1 = A_1 - A_1^1 = \frac{a_3}{2} \left(\begin{aligned} &PG_x^3 - 3PG_x^2 PG_{\min} \\ &+ 3PG_{\min}^2 PG_x - PG_{\min}^3 \end{aligned} \right) \quad (21)$$

or

$$\Delta A_1 = A_1 - A_1^1 = \frac{a_3}{2} \left(\begin{aligned} &PG_x^3 - PG_{\min}^3 - 3PG_x^2 PG_{\min} \\ &+ 3PG_{\min}^2 PG_x \end{aligned} \right) \quad (22)$$

Now from our cubic function expansion $(\alpha \pm \beta)^3$:

$$\begin{aligned} (\alpha + \beta)^3 &= (\alpha + \beta)(\alpha + \beta)^2 = (\alpha + \beta)(\alpha^2 + 2\alpha\beta + \beta^2) \\ &= \alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3 \\ &= \alpha^3 + \beta^3 + 3\alpha^2\beta + 3\alpha\beta^2 \end{aligned} \quad (23)$$

Similarly,

$$\begin{aligned} (\alpha - \beta)^3 &= (\alpha - \beta)(\alpha - \beta)^2 = (\alpha - \beta)(\alpha^2 - 2\alpha\beta + \beta^2) \\ &= \alpha^3 - 2\alpha^2\beta - \alpha\beta^2 - \alpha^2\beta - 2\alpha\beta^2 - \beta^3 \end{aligned}$$

$$= (\alpha^3 - \beta^3 - 3\alpha^2\beta + 3\alpha\beta^2) \quad (24)$$

Recalled equation (22):

$$\begin{aligned} \Delta A_1 &= A_1 - A_1^1 = \frac{a_3}{2} \\ &\quad \left(PG_x^3 - PG_{\min}^3 - 3PG_x^2 PG_{\min} + 3PG_{\min}^2 PG_x \right) \end{aligned} \quad (22')$$

$$\Delta A_1 = \alpha^3 - \beta^3 - 3\alpha^2\beta + 3\alpha\beta^2$$

Then equation (22), can be rewritten as:

$$\begin{aligned} \Delta A_1 &= A_1 - A_1^1 = \alpha^3 - \beta^3 = (\alpha - \beta)^3 = \\ &\quad \left(PG_x - PG_{\min} \right)^3 \end{aligned} \quad (25)$$

Therefore,

$$\Delta A_1 = A_1 - A_1^1 = \frac{a_3}{2} \left(PG_x - PG_{\min} \right)^3 \quad (26)$$

Case 2: 2nd Break-Point

Similarly, following the approach (from equation 1 - 26, the area under the linear-curve from PG_{\min} to PG_x and the area under the 2nd order continuous function over the same range can be obtained as:

$$\Delta A_2 = A_2 - A_2^1 = \frac{a_3}{2} \left(PG_y - PG_x \right)^3 \quad (27)$$

Case 3: 3rd Break - point

In the same manner, repeat the same process which gives as:

$$\Delta A_3 = A_3 - A_3^1 = \frac{a_3}{2} \left(PG_{\max} - PG_y \right)^3 \quad (28)$$

Evidently, it is shown from equation (26), (27) and (28) that the area difference ($\Delta A_1, \Delta A_2$ and ΔA_3) is the 'only' function of the (coefficient: a_3)

Now, substituting equation: 26, 27, and 28 into equation (5) as:

$$V = \Delta A_1^2 + \Delta A_2^2 + \Delta A_3^2 \quad (5)$$

$$\begin{aligned} V &= \frac{a_3^2}{2} \left(PG_x - PG_{\min} \right)^3 + \frac{a_3^2}{2} \left(PG_y - PG_x \right)^3 \\ &\quad + \frac{a_3^2}{2} \left(PG_{\max} - PG_y \right)^3 \end{aligned} \quad (29)$$

or

$$V = \left(\frac{a_3}{2} \right)^2 \left[\frac{\left((PG_x - PG_{\min})^3 \right)^2 + \left((PG_y - PG_x)^3 \right)^2}{\left((PG_{\max} - PG_y)^3 \right)^2} \right] \quad (30)$$

or

$$V = \frac{a^2}{4} \left[\frac{\left(PG_x - PG_{\min} \right)^6 + \left(PG_y - PG_x \right)^6}{\left(PG_{\max} - PG_y \right)^6} \right] \quad (31)$$

Now, minimizing, V with respect to PG_x and PG_y .

$$V = f(PG_x, PG_y) \quad (32)$$

This implies that, minimizing V with respect to PG_x and PG_y we take the partial derivative of the function, with respect to the points; PG_x and PG_y respectively.

for minimization of V , the conditions are:

$$\frac{\partial V}{\partial PG_x} = 0 \quad (33)$$

$$\frac{\partial V}{\partial PG_y} = 0 \quad (34)$$

Thus, conducting the partial differentiation on equation (31):

$$\frac{\partial V}{\partial PG_x} = \frac{a_3^2}{4} \left[\frac{6(PG_x - PG_{\min})^{6-1} -}{6(PG_y - PG_x)^{6-1} + 0} \right] = 0 \quad (35)$$

and

$$\frac{\partial V}{\partial PG_y} = \frac{a_3^2}{4} \left[\frac{0 + 6(PG_y - PG_x)^{6-1} -}{6(PG_{\max} - PG_y)^{6-1}} \right] = 0 \quad (36)$$

$$\frac{\partial V}{\partial PG_x} = \frac{a_3^2}{4} \left[\frac{6(PG_x - PG_{\min})^5 -}{6(PG_y - PG_x)^5 + 0} \right] = 0 \quad (37)$$

$$\frac{\partial V}{\partial PG_y} = \frac{a_3^2}{4} \left[\frac{0 + 6(PG_y - PG_x)^5 -}{6(PG_{\max} - PG_y)^5} \right] = 0 \quad (38)$$

Thus, it is evident that from equation (37) and (38)

$$PG_x - PG_{\min} = PG_y - PG_x \quad (39)$$

and

$$PG_y - PG_x = PG_{\max} - PG_y \quad (40)$$

The developed equation (39) and (40) are unique in that, the continuous 'IC' curve to be approximated by three (3) - segment of linear-curve, is such that the optimal placement arrangement at PG_x and PG_y in that order, each segment - length is equal, as shown in equation (39) and (40).

Therefore,

$$\Delta PG = PG_x - PG_{\min} = PG_y - PG_x = PG_{\max} - PG_y \quad (41)$$

$$PG_y = \frac{PG_{\max} - PG_{\min}}{3}$$

- This technique can be applied or extended to a case k -segments, in which all segments are equal to:

$$\Delta PG = \frac{PG_{\max} - PG_{\min}}{k} \quad (42)$$

- Evidently, in the case of a limiting conditions, as k approaches infinity, the linear curve also approaches the continuous (2nd order function) in incremental cost (IC) curve.

- Therefore, the area difference ($\Delta A_1, \Delta A_2, \Delta A_3, \dots$) for any segments k , can be expressed as a function of the uniform - segment length, ΔPG .

Behaviour of Generating - Unit

The behaviour of generating - units in terms of fuel consumption (coefficient: a_0, a_1, a_3) depends on either to exhibit a (concave - up) curve that is non-monotonically increasing, when $a_3 > 0$, then the extreme point is minimum, and non-monotonically increasing and it is (convex - down) curve when $a_3 < 0$, then the extreme point is maximum. It is shown in fig 2, fig. 3 and fig. 4 respectively.

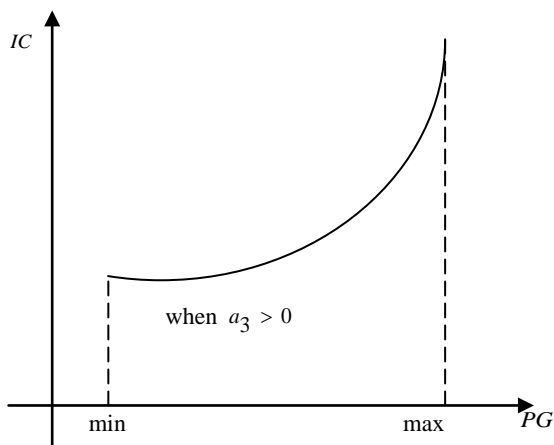


Fig.2: concave-up

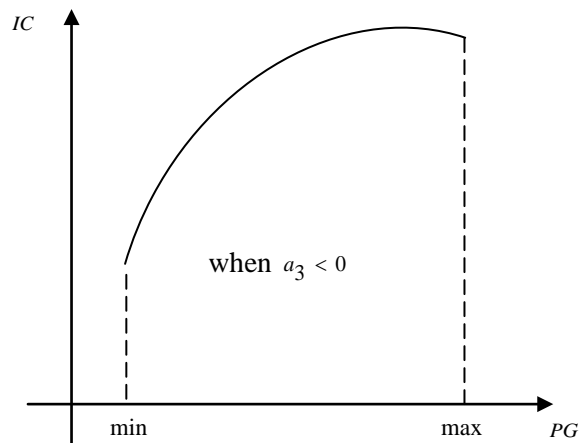


Fig.3: convex-down

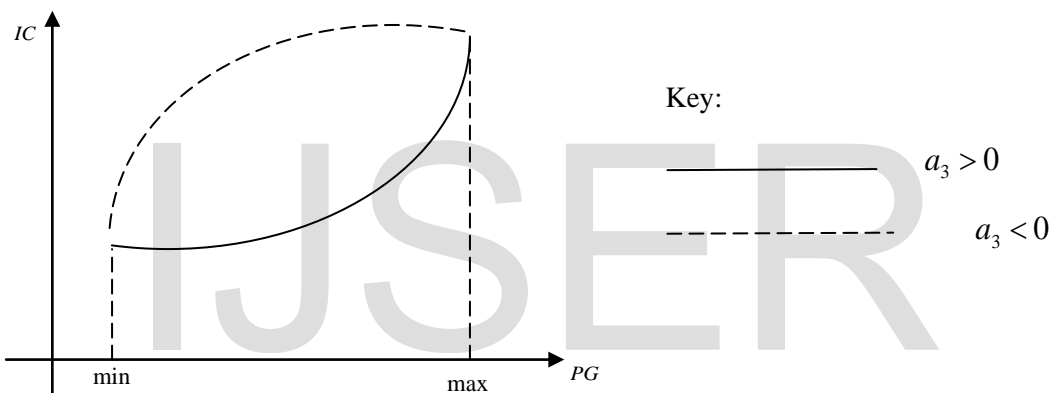


Fig. 4: Monotonically increasing

For example, when data set is collected from different generating companies, such that the behaviours of generating units exhibits 'Non-monotonically increasing - Incremental Cost (IC) curve', that is concave - up, when $a_3 > 0$ this means that the property of never increasing be adjusted to achieve the optimal fuel consumption efficiency. Therefore it is evident to adjust the fuel coefficient for these generating units, until the

incremental cost (IC) curve becomes 'Continuously - increasing. However it is assumed that the incremental fuel-cost (IC) curve for any given generator is represented as monotonically increasing linear curve with end points of each linear segment, specified by 'break point'. The data set and figures demonstrate the behaviour of generator incremental - cost curve with respect to generator output.

Break-point	Unit 1		Unit 2	
	IC	PG	IC	PG
1	5	4	4	2
2	6	12	5	10
3	9	18	6	16
4	12	20	8	22

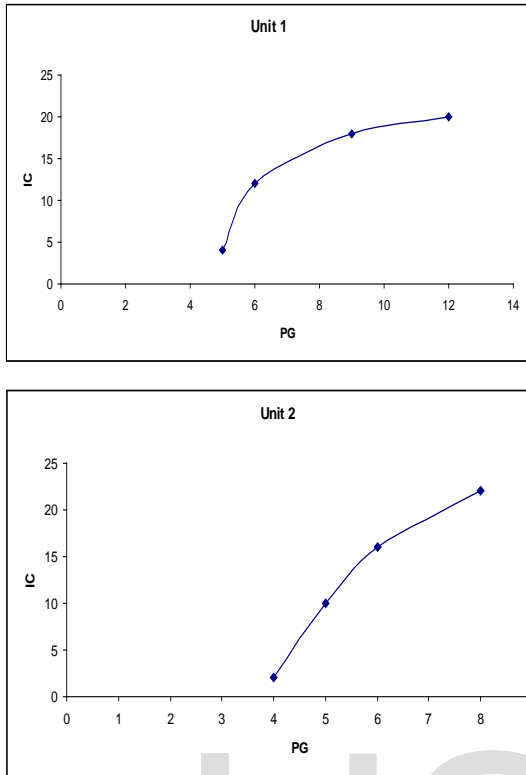


Fig. 4: Linear Incremental Cost-Curve (IC)

Therefore,

$$\Delta PG = PG_x - PG_{\min} = PG_y - PG_x = PG_{\max} - PG_y = \frac{PG_{\max} - PG_{\min}}{3} \quad (41)$$

- This technique can be applied or extended to a case of k-segments, in which all segments are equal to:

$$\Delta PG = \frac{PG_{\max} - PG_{\min}}{K} \quad (42)$$

- evidently, in the case of a limiting conditions, as approaches infinity, the linear curve, also approaches the continuous (2nd order function), incremental cost (IC) curve.
- Therefore, the area difference ($\Delta A_1, \Delta A_2, \Delta A_3, \dots$) for any segments, K, can be expressed as a function of the uniform-segment length, ΔPG .

That is,

$$\Delta Ak = \frac{a_3}{2} \Delta PG^3 = \frac{a_3}{2} \left(\frac{PG_{\max} - PG_{\min}}{m} \right)^3 \quad (43)$$

where:

$$\Delta PG^3 = \frac{PG_{\max} - PG_{\min}}{m} \quad (44)$$

Also, the total area difference expressed as:

$$\Delta A_T = \sum_{k=1}^m \frac{a_1}{2} \left(\frac{PG_{\max} - PG_{\min}}{M} \right)^3 \quad (45)$$

$$\Delta A_T = \frac{a_3}{2} (PG_{\max} - PG_{\min})^3 \left(\frac{1}{M} \right)^2 \quad (46)$$

This means that, the total area different is universally proportional to the square of M.

Conclusion

The incremental fuel cost-curve for generation expansion planning with the aim of minimizing cost of fuel is an economic problem. This paper formulated and presented a simple approximating model that need to monitor the behavior of fuel consumption pattern of the generating plants. That is, some data from generating station are collected to determine, and examine the exhibition of different curve-function, in a way to search and recommends for the best optimal point for generating stations and for an economic saving.

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